

MODELING MAGNETICALLY INSULATED POWER FLOW IN MERCURY*

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Abstract

Mercury is a 50-ns, 6-MV, 360-kA accelerator with a magnetically-insulated, inductive-voltage-adder (MIVA) architecture. The machine was formerly known as KALIF-HELIA[1] at Forschungszentrum Karlsruhe in Germany but now, with some minor modifications[2], will be sited at NRL. Mercury can be operated in either positive or negative polarity[2-4]. Voltage is added in vacuum along a magnetically insulated transmission line (MITL) from six voltage adder cells. Understanding power flow and coupling to a load in this geometry requires the application of MITL theory[5-8]. Because the electric field stresses on the cathode in the MITL exceed the vacuum explosive-emission threshold, electron emission occurs and current flow is divided between current flowing in the metal and in vacuum electron flow. This electron flow manifests itself as a loss current until the total current is large enough to magnetically insulate the emitted electrons from crossing the anode-cathode (AK) gap. Once insulated, the electrons flow axially toward the load as illustrated in Fig. 1. In particular, electron emission and flow along the MITL alters the impedance along the line and, thus, the power flow coupling between the machine and the load. The effective impedance is best described by the flow impedance, which is a function of both the geometry and the voltage. When electrons are emitted from regions having different voltages, such as in the adders or at different locations along the MITL itself, layered flow occurs, further complicating the picture. Analysis of power flow in this complex geometry is underway to understand the past performance of KALIF-HELIA and to assist in optimizing the future performance of Mercury in both polarities and for various load configurations[3,4]. The goal of this work is to develop physics-based MITL circuit-element models for the NRL transmission line code BERTHA[9] to properly treat power flow in the vacuum section of Mercury while modeling the full machine.

I. MITL MODEL

The fundamental understanding of MITL flow is derived from a pressure balance argument when space charge

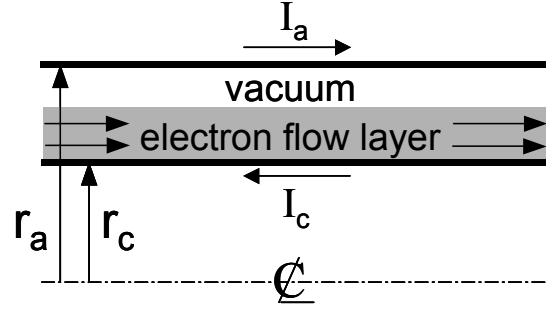


Figure 1. Schematic of MITL flow in negative polarity.

limited (SCL) emission occurs at the cathode[5-8]. This assumption of SCL emission implies that the cathode is turned-on (i.e., the cathode is a zero work function emitter) and that there is sufficient space charge in the gap to drive the electric field to zero at the cathode surface. The model allows for the possibility of additional electron emission at any point along the MITL if there is not enough space charge in the gap and electron current loss to the anode if there is too much space charge in the gap. It is also assumed that there is no ion emission from the anode, the electrons are emitted from the cathode with zero energy (and, hence, at zero pressure), and the electron pressure is negligible compared to the electromagnetic field pressure at the anode.

The general form for the flow impedance Z_f is

$$Z_f = \frac{V}{(I_a^2 - I_c^2)^{1/2}}, \quad (1)$$

where Z_0 is the vacuum impedance of the line, V is the voltage across the line, I_a is the current flowing in the anode, I_c is the current flowing in the cathode, and $I_a - I_c$ is the current flowing in the electron flow layer. This definition for Z_f is independent of the distribution of space charge in the AK gap. For the case where the space charge density is uniform across the flow layer,

$$V = Z_0 (I_a^2 - I_c^2)^{1/2} - \frac{mc^2}{2e} \frac{I_a^2 - I_c^2}{I_c^2}, \quad (2)$$

where Z_0 is the vacuum impedance, m and e are the mass and charge of an electron, and c is the speed of light. The last term is the space charge correction. These two equations can be used to determine the flow

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14. ABSTRACT Mercury is a 50-ns, 6-MV, 360-kA accelerator with a magnetically-insulated, inductive-voltage-adder (MIVA) architecture. The machine was formerly known as KALIF-HELIA[1] at Forschungszentrum Karlsruhe in Germany but now, with some minor modifications[2], will be sited at NRL. Mercury can be operated in either positive or negative polarity[2-4]. Voltage is added in vacuum along a magnetically insulated transmission line (MITL) from six voltage adder cells. Understanding power flow and coupling to a load in this geometry requires the application of MITL theory[5-8]. Because the electric field stresses on the cathode in the MITL exceed the vacuum explosive-emission threshold, electron emission occurs and current flow is divided between current flowing in the metal and in vacuum electron flow. This electron flow manifests itself as a loss current until the total current is large enough to magnetically insulate the emitted electrons from crossing the anode-cathode (AK) gap.					
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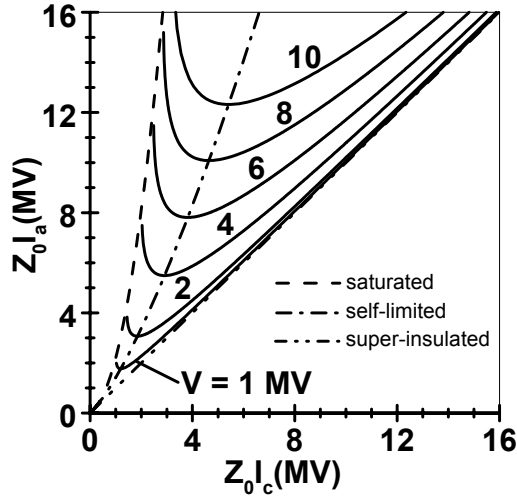


Figure 2. Plot of $Z_0 I_a$ as a function of $Z_0 I_c$ for various V .

characteristics in most situations. In the past this result has been displayed graphically by solving Eq. (2) for I_a and plotting I_a as a function of I_c for various values of V as shown in Fig. 2 [5-8]. The curves terminate at the left when the flow is saturated and the electron flow layer extends all the way to the anode. When the flow is super insulated, there is very little flow current and the flow layer is very close to the cathode. The minimum anode current for a given voltage occurs for self-limited flow, which is discussed in more detail below.

Here, expressions for the flow impedance are derived that will be needed for applying the results to BERTHA. When Z_0 , V , and I_c are known, Eq. (2) is solved for $(I_a^2 - I_c^2)^{1/2}$, which is substituted into Eq. (1) to obtain

$$Z_f(Z_0, V, I_c) = \frac{Z_0 + \left(Z_0^2 - \frac{2mc^2 V}{eI_c^2} \right)^{1/2}}{2} \quad (3)$$

Note that $Z_f = Z_0/2$ for $I_c = I_c^{\text{ins}} = (2mc^2 V / eZ_0^2)^{1/2}$, which is the minimum cathode current required for insulation (i.e., $I_c \geq I_c^{\text{ins}}$ in order for a flow solution to exist). Also note that $Z_0/2 \leq Z_f \leq Z_0$. Once Z_f is determined, I_a is found from Eq. (1) with $I_a = (I_c^2 + V^2/Z_f^2)^{1/2}$. Similarly, when Z_0 , V , and I_a are known, Eqs. (1) and (2) can be

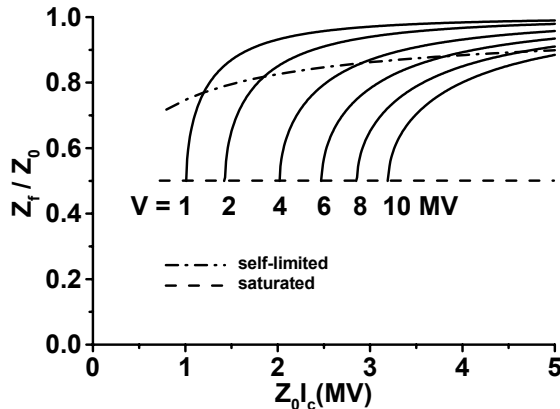


Figure 3. Plot of Z_f/Z_0 as a function of $Z_0 I_c$ for various V .

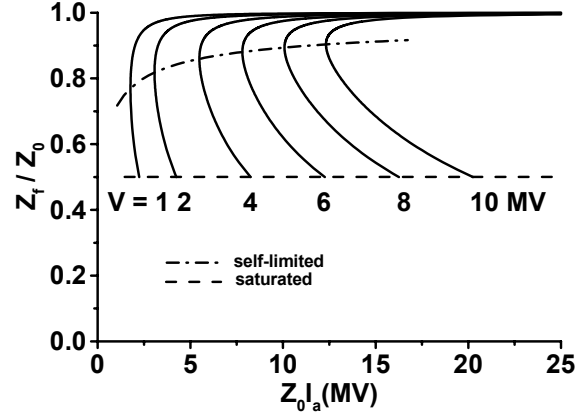


Figure 4. Plot of Z_f/Z_0 as a function of $Z_0 I_a$ for various V .

manipulated to provide

$$I_a^2 = \frac{V^2 \left(\frac{mc^2}{2eV} - I \right) Z_f + V^2 Z_0}{Z_0 Z_f^2 - Z_f^3} \quad (4)$$

where this expression needs to be inverted to find $Z_f(Z_0, V, I_a)$. Here, $I_a = I_{a0} = (2V/Z_0)(1 + mc^2/2eV)^{1/2}$ when $Z_f = Z_0/2$. Once Z_f is determined, I_c is found from Eq. (1) with $I_c = (I_a^2 - V^2/Z_f^2)^{1/2}$. Plots of the expressions for $Z_f(Z_0, V, I_c)$ and $Z_f(Z_0, V, I_a)$ are shown in Figs. 3 and 4, respectively, for various values of V . Note that $Z_f(Z_0, V, I_a)$ is double valued for $I_a^{\text{SL}} \leq I_a \leq I_{a0}$, so that care must be taken to determine which value is appropriate. Self-limited flow with $Z_f = Z_f^{\text{SL}}$ occurs at the minimum value of $I_a = I_a^{\text{SL}}$ and describes the flow that is established in a long MITL or an MITL coupled to a high impedance load (e.g., for $Z_L > Z_f^{\text{SL}}$).

Analytic expressions for the flow impedance and currents in self-limited flow have been derived for application in BERTHA and are given by

$$Z_f^{\text{SL}} = Z_0 f_{\text{SL}}(V) \quad (5)$$

$$f_{\text{SL}}(V) = \left(\frac{V_0}{V} - I \right) \pm \left[\left(\frac{V_0}{V} - I \right)^2 + \frac{I}{\left(\frac{V_0}{V} - I \right)} \right]^{1/2} \quad (6)$$

$$I_a^{\text{SL}}(V) = \frac{V}{Z_0 f_{\text{SL}}(V)} \left(\frac{I}{2 f_{\text{SL}}(V) - I} \right)^{1/2} \quad (7)$$

$$I_c^{\text{SL}}(V) = \left(\frac{V_0}{2Z_0} \right) \left(-I + \frac{20V}{V_0} + \left[I + \frac{12V}{V_0} + \frac{36V^2}{V_0^2} + \frac{32V^3}{V_0^3} \right]^{1/2} \right)^{1/2} \quad (8)$$

Here, $V_0 = mc^2/2e = 0.255$ MV, and in Eq. (6) the upper (+) sign applies when $V_0/V > 1$ and the lower (-) sign applies when $V_0/V < 1$. Note that Z_f/Z_0 , $Z_0 I_a$, and $Z_0 I_c$ are functions only of V . Plots of Z_f^{SL}/Z_0 , $Z_0 I_a^{\text{SL}}$, and $Z_0 I_c^{\text{SL}}$ are shown in Fig. 5.

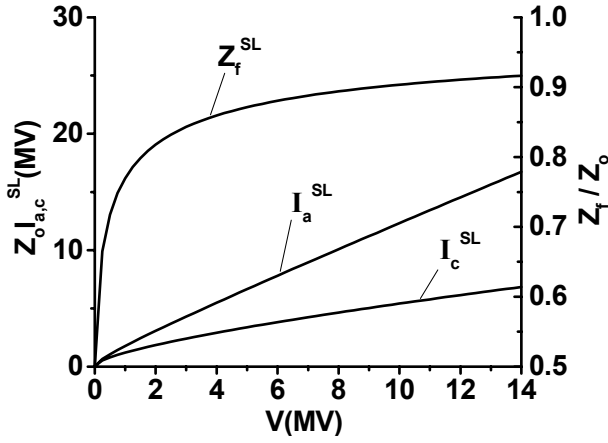


Figure 5. Plots of Z_f^{SL}/Z_0 , $Z_0 I_a^{SL}$, and $Z_0 I_c^{SL}$ as functions of V .

II. BENCHMARKING MODEL RESULTS WITH PIC SIMULATIONS

Numerical simulations of simple MITL flow problems have been run to benchmark the model. The electromagnetic hybrid/PIC code LSP [10] was used to run these simulations. A long (compared with the distance required to establish equilibrium flow) MITL that is terminated with an electron beam diode load is driven in negative polarity (i.e., the center conductor is the cathode) to equilibrium at a voltage of 1 MV. The simulation region is 10 cm long with a cathode radius of 0.5 cm and an anode radius of 1 cm, which represents a vacuum line impedance of 41.6 Ω . The model predicts a self-limited flow impedance of $Z_f^{SL} = 32 \Omega$ for this geometry at 1 MV (see Eqs. (5) and (6), or Fig. 5). Three cases were run with axial diode gaps of $D = 0.25$ cm, 0.53 cm, and 1 cm. Particle plots of these three cases are shown in Fig. 6. At $D = 0.25$ cm, the diode impedance is about 15 Ω , well below Z_f^{SL} . In this case, the equilibrium flow is on the super-insulated branch of Fig. 4 with the flow layer held close to the cathode as seen in Fig. 6a. The currents measured in the simulation at $z = 5$ cm are I_a

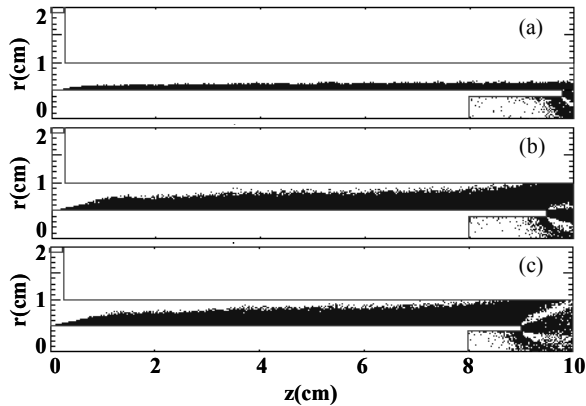


Figure 6. Plots of MITL electron flow driven at 1 MV with (a) $D = 0.25$ cm, (b) $D = 0.53$ cm, and (c) $D = 1$

$= 48.2$ kA and $I_c = 40.4$ kA, which corresponds to a flow impedance of 37.7 Ω as calculated from Eq. (1). Based on $I_a = 48.2$ kA, Eq. (4) predicts a flow impedance of 37.4 Ω and a cathode current of 40.3 kA, which are in reasonable agreement with the simulation results. This implies that the assumption of uniform charge density in the flow layer, which was used to derive Eq. (2) (and the subsequent equations), is reasonable. The charge densities measured in the simulations are compared with the assumed charge densities in Fig. 7. Although the charge density profiles found in the PIC simulations are peaked near the cathode and contain somewhat more space charge compared with the square profiles assumed in the model, the widths of the charge layers are in reasonable agreement.

For $D = 0.53$ cm, the load impedance is comparable to the self-limited flow impedance. In this simulation shown in Fig. 6b, $I_a = 44.1$ kA, $I_c = 29.7$ kA, and $Z_f = 30.6 \Omega$ at $z = 5$ cm, which are approaching to the self-limited flow values of $I_a^{SL} = 42.5$ kA, $I_c^{SL} = 28.9$ kA, and $Z_f^{SL} = 32 \Omega$ predicted by the model. For $D = 1$ cm as shown in Fig 6c, the load impedance is significantly larger than the Z_f^{SL} and the MITL is expected to run self-limited. The simulation values of $I_a = 43.1$ kA, $I_c = 28.5$ kA, and $Z_f = 30.6 \Omega$ are in reasonable agreement with the predicted self-limited flow values.

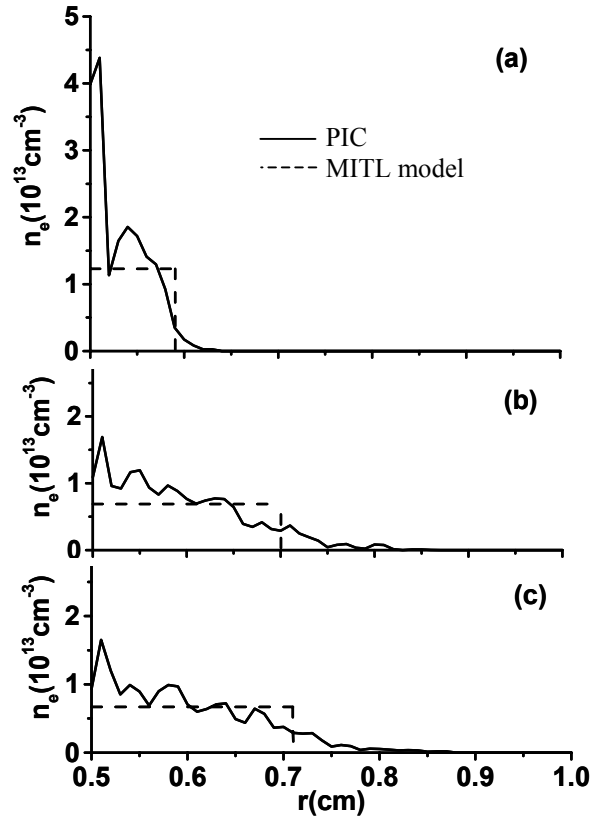


Figure 7. $n_e(r)$ from PIC simulations with (a) $D = 0.25$ cm, (b) $D = 0.53$ cm, and (c) $D = 1$ cm.

III. INCORPORATING THE MITL MODEL INTO BERTHA

The TRIFL code was developed previously to model the MITL flow of pulsed power vacuum transmission systems[11]. The purpose of the work presented here is to incorporate the MITL model into a transmission line code in order to develop the capability to model a complete system from the primary energy store to the load including an MITL section. The transmission line code BERTHA will be used for this purpose[9]. An algorithm for incorporating the MITL model into BERTHA needs to be flexible enough to handle a number of important processes. The MITL is modeled as a series of short, one-time-step-long, variable impedance transmission line elements each with a variable shunt resistance. The impedance of each element is set at the instantaneous local flow impedance and the shunt resistance is adjusted to account for losses in the insulation front, at impedance transitions, and in the load-coupling region. Where there are no losses, the shunt resistance is set to a large value.

The algorithm must operate in either positive or negative polarity and account for voltage reversal, while handling cathode turn-on and emission in the insulation front, current loss to the anode in overmatched impedance transitions, addition to the flow current in undermatched impedance transitions, voltage addition at adder junctions with and without flow in the adders, self-limited flow when appropriate, re-trapping, and flow coupling to a load. At each time step, the voltage waves on the transmission line are advanced and then the impedance and shunt resistance of each element are adjusted to account for the change in flow conditions.

Once turn-on occurs, electron emission, limited to the SCL current density, launches flow into the MITL section. The flow impedance is calculated for each MITL element starting from the upstream end of the MITL working downstream to the load. When a downstream MITL element is overmatched to its upstream neighbor, current is lost to the anode but the cathode current is preserved across the transition. In this case, V and the downstream I_c are known and Eq. (3) (or Fig. 3) is used to find the downstream Z_f . When a downstream MITL element is undermatched to its upstream neighbor, additional current is launched into the flow but the anode current is preserved across the transition. In this case, V and the downstream I_a are known and Eq. (4) (or Fig. 4) is used to find the downstream Z_f . Matching conditions at transitions where adders connect to the MITL have also been derived in order to apply Eq. (3) or (4) at these junctions. At the load, the flow current combines with the load current or is lost to the anode depending on the load properties. For low impedance loads the flow in the MITL is determined by the load impedance, while for high impedance loads self-limited flow is established.

This algorithm for incorporating the MITL model into BERTHA has been developed and is being implemented.

IV. SUMMARY

A model has been developed to describe MITL flow in a vacuum transmission line section of pulsed power generators for implementation in the transmission line code BERTHA. The immediate application will be analysis of power flow in the new Mercury accelerator at NRL, which is based on MIVA technology. The model has been successfully benchmarked against basic PIC simulations and the algorithm for applying the model is now being incorporated into BERTHA.

V. REFERENCES

- [1] P. Hoppe, J. Singer, H. Bluhm, K. Leber, D. Rusch, and Otto Stoltz, "Energy balance of the TW pulsed power generator KALIF-HELIA," Proc. 13th Intl. Pulsed Power Conf., Las Vegas, NV, June 17-22, 2001, pp. 596-599.
- [2] R.J. Commisso, R.J. Allen, G. Cooperstein, R.C. Fisher, D.D. Hinshelwood, D.P. Murphy, J.M. Neri, P.F. Ottinger, D.G. Phipps, J.W. Schumer, O. Stolz, K. Childers, V. Bailey, D. Creeley, S. Drury, J. Kishi, H. Nishimoto, I. Smith, P. Hoppe, and H.J. Bluhm, "Status of the Mercury pulsed power generator, a 6-MV, 360-kA magnetically-insulated inductive voltage adder," these proceedings.
- [3] J.W. Schumer, R.J. Allen, P.F. Ottinger, and R.J. Commisso, "Coupling power-flow from the Mercury MIVA into a rod-pinch diode," these proceedings.
- [4] R.J. Allen, P.F. Ottinger, R.J. Commisso, J.W. Schumer, P. Hoppe, and I. Smith, "Electrical modeling of Mercury for optimal machine design and performance estimation," these proceedings.
- [5] C.W. Mendel, D.B. Seidel, and S.E. Rosenthal, "A simple theory of magnetic insulation from basic physical considerations," *Laser and Part. Beams* **1**, 311-320, 1983.
- [6] C.W. Mendel and S.E. Rosenthal, "Modeling magnetically insulated devices using flow impedance," *Phys. Plasmas* **2**, 1332-1342, 1995.
- [7] C.W. Mendel and S.E. Rosenthal, "Dynamic modeling of magnetically insulated transmission line systems," *Phys. Plasmas* **3**, 4207-4219, 1996.
- [8] S.E. Rosenthal, "Characterization of Electron Flow in Negative- and Positive-Polarity Linear-Induction Accelerators," *IEEE Trans. Plasma Sci.* **19**, 822-830, 1991.
- [9] D.D. Hinshelwood, "BERTHA - a versatile transmission line and circuit code," NRL Memorandum Report 5185, November 1983.
- [10] D.R. Welch, D.V. Rose, B.V. Oliver, and R.E. Clark, "Simulation techniques for heavy ion fusion chamber transport," *Nucl. Instrum. Methods Phys. Res. A* **464**, 134-139 (2001).
- [11] C.W. Mendel, Jr., S.E. Rosenthal, and D.B. Seidel, "Modeling pulsed power vacuum transmission systems," Sandia National Laboratories, September 14, 1999, unpublished.